Resource control in sequent lambda calculus

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Outline of the talk

1. Logic vs Computation
2. From λ towards sequent λ calculi
3. Sequent λ calculus with resource control

Outline

1. Logic vs Computation
2. From λ towards sequent λ calculi
3. Sequent λ calculus with resource control

Computational interpretations of intuitionistic logic

- 1950s Curry
- 1968 (1980) Howard formulae-as-types
- 1970s Lambek - CCC Cartesian Closed Categories
- 1970s de Bruijn AUTOMATH

logic vs term calculus

$$\vdash A \quad \Leftrightarrow \vdash t : A$$

Curry - Howard - Lambek - de Bruijn correspondence

formulae – as – types
proofs – as – terms
proofs – as – programs

Computational interpretations of intuitionistic logic

Logic vs *term calculi*

- Axiomatic (Hilbert) system (axioms/Modus Ponens)
  Combinatory Logic (combinators/application)
  1930s Schönfinkel, Curry
- Natural Deduction (introduction/elimination)
  λ *calculus(abstraction/ application)*
  1940s Church
- Sequent Calculus (right/left introduction/cut)
  various attempts λ *calculus*
  (abstraction/application/substitution)
  1970s

Outline

Sequent calculus intuitionistic logic

- Pottinger, Zucker 1970s comparing cut-elimination to proof normalization
- Gallier [1991]
- Mints [1996]
- Barendregt, Ghilezan [2000]: λLJ-calculus

But in these, terms do not encode derivations.

- Herbelin [1995]: $\bar{\lambda}$-calculus - developed the idea of making terms explicitly represent sequent calculus derivations.
- Dyckoff and Pinto [1998]
- Computation over terms reflects cut-elimination
- Espírito Santo [2006]: λ^{Gtz}-calculus

λLJ-calculus

- type system LJ - sequent types structure
- term calculus λ-calculus

$$\frac{}{\Gamma \quad A \vdash \quad A}\,(axiom) \qquad \frac{\Gamma \vdash \quad A \quad \Gamma, \quad B \vdash \quad C}{\Gamma, \quad A \to B \vdash \qquad C}\,(\to_{left})$$

$$\frac{\Gamma, \quad A \vdash \quad B}{\Gamma \vdash \qquad : A \to B}\,(\to_{right}) \qquad \frac{\Gamma \vdash \quad A \quad \Gamma, \quad A \vdash \quad B}{\Gamma \vdash \qquad : B}\,(cut)$$

λLJ-calculus

- type system LJ - sequent types structure
- term calculus λ-calculus - natural deduction term structure

$$\frac{}{\Gamma x : A \vdash x : A}\ (axiom) \qquad \frac{\Gamma \vdash t : A \quad \Gamma, x : B \vdash s : C}{\Gamma, y : A \to B \vdash s[x := yt] : C}\ (\to_{left})$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash (\lambda x.t) : A \to B}\ (\to_{right}) \qquad \frac{\Gamma \vdash t : A \quad \Gamma, x : A \vdash s : B}{\Gamma \vdash s[x := t] : B}\ (cut)$$

From λLJ to $\bar{\lambda}$-calculus

λLJ-calculus, Barendregt et al. [2010]

- Using a subsystem λLJ^{cf} Gentzen's Hauptsatz (cut-elimination) theorem is easily proved!
- But, the Curry-Howard correspondence fails...

$$\begin{array}{ccccc} u & \mapsto & yz & \mapsto & \lambda x.yz \\ u & \mapsto & \lambda x.u & \mapsto & \lambda x.yz. \end{array}$$

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$\bar{\lambda}$-calculus of Herbelin

- introduction of explicit substitution

$$\lambda x.(u\langle u = yz\rangle) \qquad (\lambda x.u)\langle u = yz\rangle$$

- restriction of the sequent logic LJ - LJT:
  $(\Gamma;\ \vdash A)$ i $(\Gamma; B \vdash A)$
- introduction of a new constructor - list of arguments

λ^{Gtz}-calculus

Syntax

$$\begin{array}{llcl} \text{(terms)} & t, u, v & ::= & x \mid \lambda x.t \mid tk \\ \text{(contexts)} & k & ::= & \widehat{x}.t \mid u :: k \end{array}$$

Reductions

$$\begin{array}{lrcl} (\beta) & (\lambda x.t)(u :: k) & \to & u\widehat{x}.(tk) \\ (\pi) & (tk)k' & \to & t(k@k') \\ (\sigma) & t\widehat{x}.v & \to & v\langle x := t\rangle \\ (\mu) & \widehat{x}.xk & \to & k, \ \text{ako } x \notin k \end{array}$$

- $v\langle x := t\rangle$ is a meta-substitution;
- $k@k'$ is an append

λ^{Gtz} - simple types

Types:

$$A, B \;::=\; p \mid A \to B$$

Type assignments:

- $\Gamma \vdash t : A$ - for terms;
- $\Gamma; B \vdash k : A$ - for contexts

$$\frac{}{\Gamma, x : A \vdash x : A}\,(Ax) \qquad \frac{\Gamma, x : A \vdash t : B}{\Gamma; A \vdash \widehat{x}.t : B}\,(Sel)$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x.t : A \to B}\,(\to_R) \qquad \frac{\Gamma \vdash t : A \quad \Gamma; B \vdash k : C}{\Gamma; A \to B \vdash t :: k : C}\,(\to_L)$$

$$\frac{\Gamma \vdash t : A \quad \Gamma; A \vdash k : B}{\Gamma \vdash tk : B}\,(Cut)$$

Properties

- Subject reduction
- Strong normalisation property (Typeability implies SN)
- Characterisation of strong normalisation with intersection types (SN implies typeability)

  Typeability $\Leftrightarrow$ SN

  [Espirito Santo, S.G., Ivetić, Likavec]

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  $$\text{Typeability} \Leftrightarrow \text{SN}$$

  [Espirito Santo, S.G., Ivetić, Likavec]
- ΛJ - generalised application [Matthes]
- λx - explicit substitutions [Rose and Bloo]
- $\lambda x \cap$ - with intersection types [S.Lengrand et al.]

Outline

Sequent calculus - LJ
EXPLICIT structural rules

$$\frac{}{A \vdash A}\,(Ax)$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B}\,(\rightarrow_R) \qquad \frac{\Gamma \vdash A \qquad \Delta, B \vdash C}{\Gamma, \Delta, A \rightarrow B \vdash C}\,(\rightarrow_L)$$

$$\frac{\Gamma \vdash A \qquad \Delta, A \vdash B}{\Gamma, \Delta \vdash B}\,(Cut)$$

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B}\,(Weak) \qquad \frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}\,(Cont)$$

Implicit structural rules

- contexts are sets
- Axiom: $\Gamma, A \vdash A$
- context-sharing rules

Explicit structural rules

- contexts are multisets
- Axiom: $A \vdash A$
- context-splitting rules

Sequent lambda calculus with resource control- ®$\underline{\lambda}$

- Derived from λ^{Gtz} by adding explicit operators for weakening and contraction
- Inspired by λlxr-calculus [Kesner and Lengrand, 2005]
- Terms control resources:
  1. every variable occurs (at most) once
  2. every binder does bind an occurrence of a free variable

Example

$I = \lambda x.x$

$K = \lambda xy.x$

$\Delta = \lambda x.x(x :: \widehat{z}.z) \qquad (\lambda x.xx)$

$W = \lambda xy.x(y(y :: \widehat{z}.z) :: \widehat{z}.z) \qquad (\lambda xy.xyy)$

The syntax of ®$\underline{\lambda}$

Values	T	$::=$	$x \mid \lambda x.t \mid x \odot t \mid x <^{x_1}_{x_2} t$
Terms	t	$::=$	$T \mid tk$
Contexts	k	$::=$	$\widehat{x}.t \mid t :: k \mid x \odot k \mid x <^{x_1}_{x_2} k$

New features:

- new constructors for weakening ($x \odot e$) and contraction($x <^{x_1}_{x_2} e$)
- value - a new syntactic category for regaining confluence

Example

$I = \lambda x.x$	
$K = \lambda xy.x$	$\lambda xy.y \odot x$
$\Delta = \lambda x.x(x :: \widehat{z}.z)$	$\lambda x.x <^{x_1}_{x_2} (x_1(x_2 :: \widehat{z}.z))$
$W = \lambda xy.x(y(y :: \widehat{z}.z) :: \widehat{z}.z)$	$\lambda xy.x(y <^{y_1}_{y_2} (y_1(y_2 :: \widehat{z}.z) :: \widehat{z}.z)$

Reduction rules of ®$\underline{\lambda}$

Reductions of λ^{Gtz}:

$$\begin{array}{lrcl} (\beta) & (\lambda x.t)(u :: k) & \to & u(\widehat{x}.tk) \\ (\pi) & (tk)k' & \to & t(k@k') \\ (\mu) & \widehat{x}.xk & \to & k \end{array}$$

Substitution: (σ) is divided into several (5) reductions

Contraction propagation:

$$(\gamma_1) \quad x <^{x_1}_{x_2} (\lambda y.t) \quad \to \quad \lambda y.x <^{x_1}_{x_2} t$$

Weakening extraction:

$$(\omega_1) \quad \lambda x.(y \odot t) \quad \to \quad y \odot (\lambda x.t), \quad x \neq y$$

Simply typed ®$\underline{\lambda}$

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$$\frac{\Gamma, x : A, y : A \vdash t : B}{\Gamma, z : A \vdash z <^{x}_{y} t : B}\ (Cont_t) \qquad \frac{\Gamma \vdash t : B}{\Gamma, x : A \vdash x \odot t : B}\ (Weak_t)$$

$$\frac{\Gamma, x : A, y : A; C \vdash k : B}{\Gamma, z : A; C \vdash z <^{x}_{y} k : B}\ (Cont_k) \qquad \frac{\Gamma; C \vdash k : B}{\Gamma, x : A; C \vdash x \odot k : B}\ (Weak_k)$$

The properties of $\text{®}\underline{\lambda}$

- Curry Howard correspondence with **BCKW** and substructural logics **BCK**, **BCW**

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- *Lambek calculus - associativity* ***B****, commutativity* ***C****?*

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- Curry Howard correspondence with **BCKW** and substructural logics **BCK**, **BCW**
- *Lambek calculus - associativity **B**, commutativity **C**?*
- Confluence - parallel reductions technique (Takahashi)
- Subject reduction - typing rules syntax directed
- Strong normalization - rewriting techniques

The properties of ®$\underline{\lambda}$

- Curry Howard correspondence with **BCKW** and substructural logics **BCK**, **BCW**
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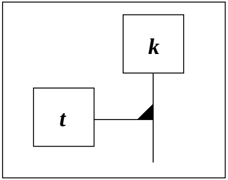
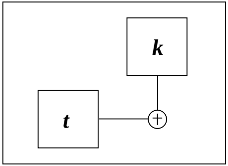
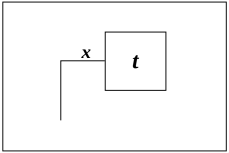
- Confluence - parallel reductions technique (Takahashi)
- Subject reduction - typing rules syntax directed
- Strong normalization - rewriting techniques

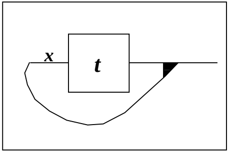
- *Characterization of strong normalization?*
- *Diagrammatic representations?*

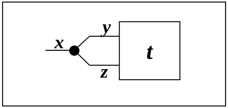
Ongoing - Diagrams for ®$\underline{\lambda}$

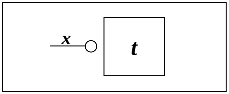
Cons: 

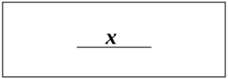
Application: 

Selection: 

Abstraction: 

Contraction: 

Weakening: 

Variable: 

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